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ABSTRACT

A scaling procedure has been developed for ordering binary parallelogram preference data. The procedure uses minimum variance of the item ranks averaged across persons as the optimization criterion. Two seriation strategies are employed. One is pairwise interchange. The second joins together the vector end points and breaks this circle between each pair of points. The two strategies are alternated until the minimum variance across persons is found. The procedure has been used successfully on artificial data, attitudinal data collected by the authors, and previously published archeological data. The procedure is an improvement on alternate methods in that it uses all of the information in the data and provides easily interpretable results. Three attitudinal scales are included. (Author)

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Parallelogram Scaling of Binary Items

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Abstract

A scaling procedure has been developed for ordering binary parallelogram preference data. The procedure uses minimum variance of the item ranks averaged across persons as the optimization criterion. Two seriation strategies are employed. One is pairwise interchange. The second joins together the vector end points and breaks this circle between each pair of points. The two strategies are alternated until the minimum variance across persons is found.

The procedure has been used successfully on artificial data, attitudinal data collected by the authors, and previously published archeological data. The procedure is an improvement on alternate methods in that it uses all of the information in the data and provides easily interpretable results.

Parallelogram Scaling of Binary Items

Parallelogram Data

Most one-dimensional test and questionnaire data is assumed to approximate a "cumulative" item characteristic function. This means that if items and persons are correctly ordered, then the pattern of positive responses in the score matrix is triangular. The prototype here is a Guttman scale.

Sometimes we are presented with data that fit what Coombs (1964) calls a parallelogram model. Parallelogram data differ from Guttman scale data in which endorsement of an item implies endorsement of all items "to the left" of the endorsed item. Parallelogram data arises when a subject's ideal point is embedded in a band of items that can occur anywhere on a continuum. Guttman data can come from responses to achievement test items when the items are of increasing difficulty and the persons are of increasing ability. In contrast, parallelogram data would occur if subjects were asked to endorse weather conditions according to their preference for that particular type of weather. Assuming that there are people who like weather that is at each point on the continuum from stormy to sunny, the correctly ordered data would assume a parallelogram shape--a diagonal band of 1's across the middle with triangles of 0's in the lower left and upper right corners.

People preferring moderate weather do not also prefer extreme weather of either kind, whereas people passing a medium item on a test would be expected to pass an easy one. Figure 1b shows an error-free person by item response matrix in parallelogram form. The prototype is a Thurstone attitude scale rather than a Guttman attitude scale.

Reordering to Achieve Parallelogram Form

Recently we have been developing a method for analyzing such data to recover the proper order for items and persons. The strategy is to organize items so they achieve as tight as possible a diagonal band of positive responses with the outside triangles as empty as possible.

When the items are in the correct order, each person's positive responses will be tightly bunched. The more compact the array of 1's, the smaller the variance of the ranks of the endorsed items. The criterion we have chosen to optimize is the variance of the ranks of the items endorsed by each person, averaged across persons. The procedure we are using first computes the average within person rank-variance for the unordered person by item matrix. Then it finds the two items which, when interchanged, will achieve the greatest reduction in variance. The procedure is iterative and continues until no pairwise interchange will produce a reduction of variance. The final column order is then applied to the

data matrix and an ordered matrix can be produced. The process can then be applied to the rows of the matrix to achieve an optimum person order.

A perfect 10x10 parallelogram matrix was scrambled across columns and used to illustrate this procedure. The unordered matrix appears in Figure 1a. The iteration history is shown in Figure 1c. After 14 iterations the original average variance of 5.27 was reduced to 0.56, the minimum possible value, and the correct rearrangement was found, as shown in Figure 1b.

The procedure has also been applied to real data. One set consisted of responses to attitude scale items. We constructed a questionnaire of 147 items randomly ordered from five attitude scales. The scales were Self-esteem, Tolerance for Ambiguity, War, Politics, and Internal/External locus of control. The 90 subjects (psychology undergraduates at the University of Southern California) indicated their endorsement of each item on a 4-pt. scale. The data were dichotomized, 1's and 2's becoming 1's, indicating agreement, and 3's and 4's becoming 0's, indicating disagreement. The responses to each scale were then analyzed separately. In most cases, the application of the procedure gave results that seemed highly plausible as an order for the items.

However, a number of things came to light through analysis of these real data that were not apparent with the small, perfect matrix. The 13 items in the self-esteem scale were reasonably ordered except for items 4 and 5 ("I feel I can't do anything right," and "I feel I do not have much to be proud of.") which were at the most positive end of the scale next to item 11 ("I am always the best at everything I try."). The pairwise interchange strategy was unable to move the items because there was no single item with which to interchange either one of them that would improve the average variance criterion. In light of this, we developed a second reordering algorithm. Picture the list of items as a chain of beads. The chain of items is joined at the ends into a circle and then popped open between each pair of items. The variance is computed for each possible vector of item orderings and the ordering with the smallest variance is sent back to the pairwise interchange procedure. The two procedures alternate until neither will produce a reduction in variance. The item orders that resulted for the Self-esteem, Ambiguity, and War scales are given in Tables 1-3, and they seem very satisfactory from the point of view of content, representing a progression along a continuum.

In principle, the method can be applied to both dimensions of the matrix although it becomes time-consuming if the order of the data matrix is large. We have used

this approach on a set of archeological data from Hobson, Kendall, and Tautu (1971). The rows represent 70 artifact types and the columns 59 tombs. The ordered data matrix appears in Figure 2. The data were input in the order that Hobson, et al found using their ordering procedure. We were able to reduce the average variance of the artifacts (rows) from 32.56 to 18.82 and the average variance of the graves (columns) from 12.19 to 8.41 using our procedure.

Where one dimension of the data matrix is large, this approach can be time-consuming. As an alternative, the larger dimension of the matrix can be ordered in terms of the mean or median rank of endorsed items.

Rational Starting Order

Our procedure and philoscphy has been to treat only the rank order of the items or persons, thus minimizing the variance of these integer-valued variables. If the idea is to provide a more general scale for the items or persons, one that is not limited to consecutive integers, then the present problem relates directly to several others that have a long history. In particular, minimizing tha variance within rows is equivalent to maximizing the variance between rows. If the matrix of dichotomous data is X, then the problem is seen to be equivalent to the "method of reciprocal averages" and all its synonyms. See Heiser(1981, p. 64 et seq.) for a discussion and

integration. The solution vector for the column scale is then an eigenvector of a matrix related to X'X, and the one for the row scale is related similarly to XX'. Thus this eigenvector can serve as a starting position for the present ordinal approach, thus reducing the number of iterations necessary. This has uncovered the fact that the final ordering is sensitive to the starting order as the eigenvector starting order can produce a different result, sometimes with a higher variance, than that found with a random starting order. In our political scale of 47 items, the eigenvector starting order produced a final order with average variance of 121.3, whereas the random input order produced a final order with average variance of 95.8. The war scale using the eigenvector starting order has a final variance of 8.8; using the random order, the variance became 8.3.

Extensions and Generalizations

We have also explored dichotomizing the responses by cutting the 4-point response scale at each point. This is easily handled by treating each resulting X matrix as a replication, and analyzing them all consecutively.

An area we are exploring is the identification of "bad" items. Substantively, these would be items that do not belong in the scale. They are items that subjects respond to inconsistently compared to the pattern of responses on

other items in the scale.

Using Coombs' (1964) terminology, the data we have explored is pick k/n data. Order k/n data is also possible. Parallelogram data is inherently interesting but there are not readily available programs to analyze it. Typically, researchers throw away mid-range items and recode the negative items (or vice versa). Or, multidimensional scaling (MDS) is attempted. We have obtained 2-dimensional MDS solutions using ALSCAL (Young and Lewyckyj, 1980), and in some cases found agreement with our results, but the MDS solutions required much more subjective interpretation than our procedure did. Our procedure makes it possible for a researcher to retain all of the information and find an interpretable and useful scale for parallelogram data.

REFERENCE LIST

1. Coombs, C.H. A Theory of Data. New York: John Wiley and Sons, 1964.

2. Heiser, W.J. Unfolding Analysis of Proximity Data. The Netherlands: Reprodienst Psychologie RUL, 1981.

3. Hodson, R.R., Kendall, D.G., and Tautu, P., eds. Mathematics in the Archeological and Historical Sciences. Edinburgh: Edinburgh University Press, 1971.

4. Young, F. W. and Lewyckyj, R. Alscal-4 User's Guide., 2nd Ed. Chapel Hill, North Carolina: University of North Carolina, 1980.

Table 1

Self-Esteem Scale

4. I feel I can't do anything right.
5. I feel I do not have much to be proud of.
12. Sometimes I think I'm no good at all.
9. I'm not able to do some things as well as most people.
3. I take a positive attitude towards myself.
6. I feel I have a number of good qualities.
2. I feel I have many good qualities
1. I am a useful person to have around.
8. I feel I am a person of worth, at least on a par with others.
13. When I do a job, I do it well
7. I am able to do things better than most people.
10. I am usually the one who can do things best.
11. I am always the best at everything I try.

Table 2

Ordered Ambiguity Scale

24. An expert who doesn't come up with a definite answer probably doesn't know too much.

2. The sooner we all acquire similar values and ideals the better.

1. A person who leads an even, regular life in which few surprises or unexpected happenings arise, really has a lot to be grateful for.

10. My best parties are ones where I already know everyone.

8. I like everything to be predictable and certain.

3. I don't like to undertake any project unless I have a pretty good idea as to how it will turn out.

20. What we are used to is always preferable to what is unfamiliar.

16. Sometimes I like travelling with no particular destination in mind.

4. Teachers or supervisors who hand out vague assignments give a chance for one to show initiative and originality.

21. In the long run it is possible to get more done by tackling small, simple problems rather than large and complicated ones.

22. A good job is one where what is to be done and how it is to be done are always clear.

7. I like parties where I know most of the people more than ones where all or most of the people are complete strangers.

5. I often wish people would be more definite about things.

18. Although I generally like to know how things are going to turn out, I realize that some things in life are unpredictable.

12. Variety is the spice of life.

23. Often the most interesting and stimulating people are those who don't mind being different and original.

13. Sometimes a little surprise can break up the monotony of routine.

17. Knowing what's going to happen all the time is boring.

19. What I look forward to at social functions is meeting new people.

25. A good teacher is one who makes you wonder about your way of looking at things.

Ordered Ambiguity Scale continued

29. I would like to live in a foreign country for a while.

11. It is more fun to tackle a complicated problem than to solve a simple one.

26. What I really enjoy about life are all the surprises.

15. Many of our most important decisions are based upon insufficient information.

14. I often like to start projects even if I don't have a clear idea of the final result.

9. People who insist upon a yes or no answer just don't know how complicated things really are.

30. It's much more fun doing things on the spur of the moment than planning in advance.

6. There is really no such thing as a problem that can't be solved.

27. People who fit their lives to a schedule probably miss most of the joy of living.

28. I really like not knowing what to expect.

Table 3

Ordered War Scale

14. I would immediately go to war and would do everything in my power to influence others to do the same.

6. I would participate in war to defend a foreign country against its enemy.

2. In time of war I would disregard any possible exemptions and enlist immediately.

3. In time of war I would not enlist but would give whatever financial aid I could.

15. In time of war I would offer my services in whatever capacity I could.

4. In time of war I would support my country even against my convictions.

1. I would participate in a war only if I felt the cause was just.

7. In time of war, I would wait a month or two if possible before I would enlist.

16. I would fight in a war only if my country were invaded by the enemy.

13. I would not go to war unless I were drafted.

11. I would immediately attempt to find some technicality on which to evade going to war.

5. I would participate in war only if my family were being threatened.

8. I would refuse to participate in any way in war.

9. I would rather be called a coward than go to war.

17. I would not only refuse to participate in any way in war but also attempt to influence public opinion against war.

10. I would take part in war only to avoid social ostracism.

12. I would go to war only if my friends went to war.

a.

Persons

Items

0	1	1	0	0	0	0	0	0	1
0	1	1	0	0	1	0	0	0	0
0	0	1	0	0	1	0	0	1	0
0	0	0	1	0	1	0	0	1	0
1	0	0	1	0	0	0	0	1	0
1	0	0	1	0	0	1	0	0	0
1	0	0	0	1	0	1	0	0	0
0	0	0	0	1	0	1	1	0	0
0	0	0	0	1	0	0	1	0	0
0	0	0	0	0	0	0	1	0	0

b.

Items

1	1	1	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0	0
0	0	1	1	1	0	0	0	0	0
0	0	0	1	1	1	0	0	0	0
0	0	0	0	1	1	1	0	0	0
0	0	0	0	0	1	1	1	0	0
0	0	0	0	0	0	1	1	1	0
0	0	0	0	0	0	0	1	1	1
0	0	0	0	0	0	0	0	1	1
0	0	0	0	0	0	0	0	0	1

c.

Iteration	Variance	Improvement	Pair	
1	5.269444444	1.8	1	10
2	3.465444444	1.533333333	4	8
3	1.936111111	0.1777777778	4	9
4	1.758333333	0.05633333333	5	6
5	1.7	0.0888888888	7	9
6	1.511111111	0.09722222222	6	8
7	1.513888885	0.03611111111	5	9
8	1.477777778	0.05888888887	4	7
9	1.411111111	0.09722222222	8	9
10	1.313888888	0.05833333333	4	5
11	1.255555556	0.3555555556	1	9
12	0.9	0.1777777778	1	4
13	0.7222222222	0.1638888889	5	7
14	0.5583333333	0	1	1

Figure 1